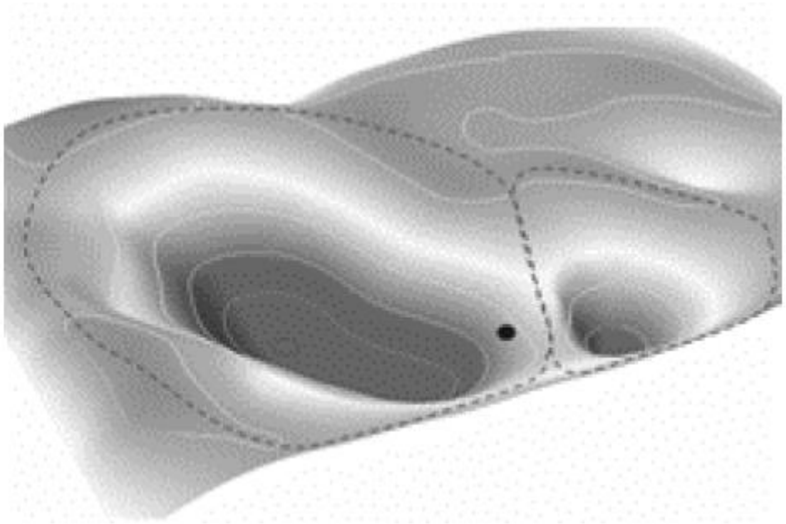
Kicks and flows:
A dynamical systems approach to modeling resilience



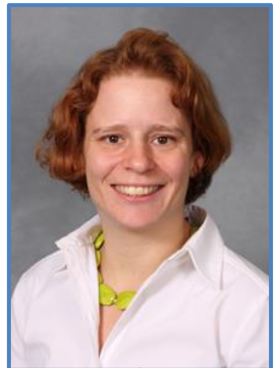
Kate Meyer,

September 23, 2014

Image: Walker et al. (2004)

Collaborators

Sarah Iams



Mary Lou Zeeman

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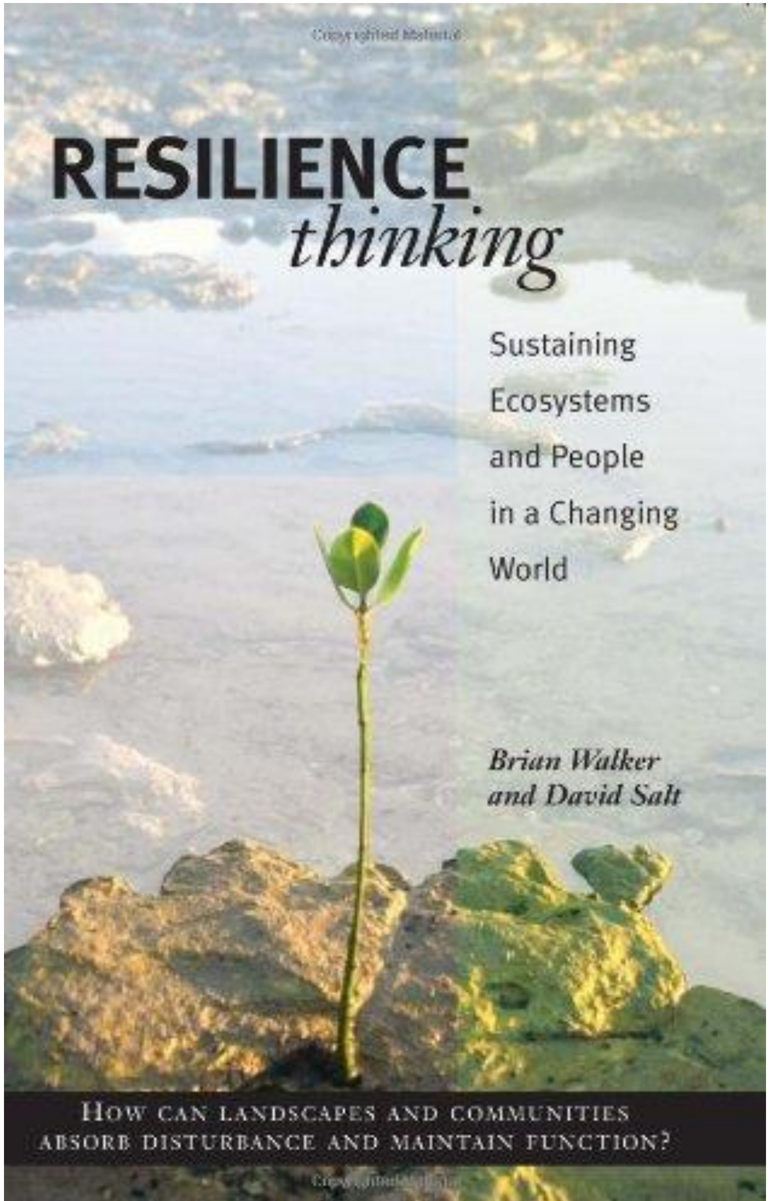
Stephen Lightenberg

Kicks and flows:
A dynamical systems approach to modeling resilience

Outline

1. The resilience paradigm
2. Defining resilience mathematically
3. A kick-flow system
    a. Behavior in a linear system
    b. Experiments in 2D nonlinear systems
4. Future Directions

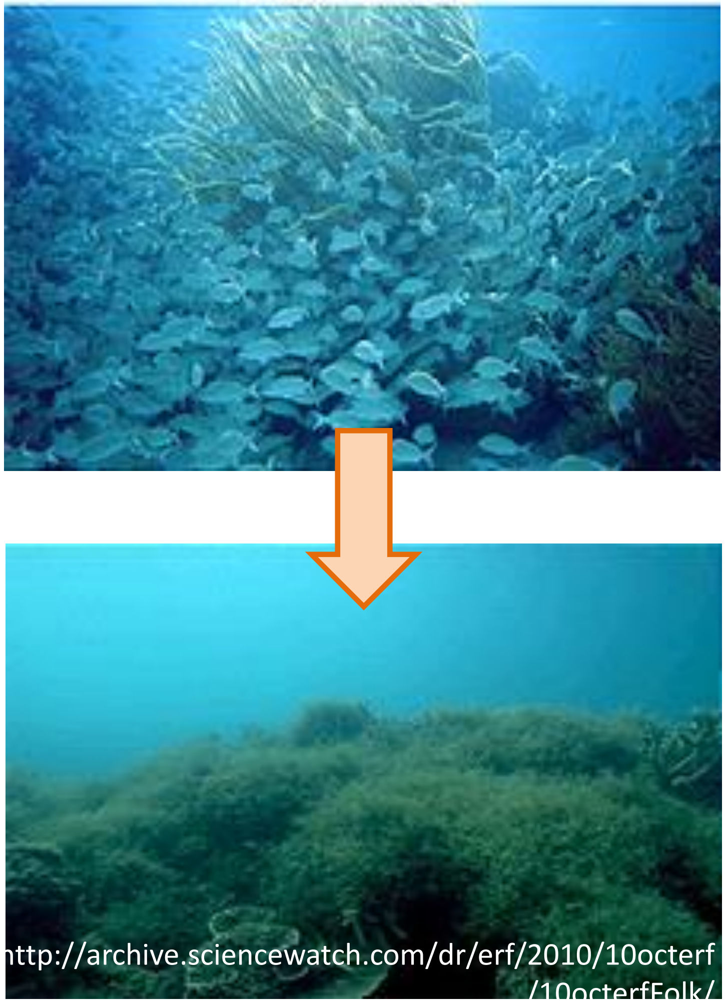
1. The resilience paradigm



Resilience …

the ***capacity*** *of [a] system*
to ***absorb change*** *and disturbances*
and still ***retain*** *its*
basic ***structure and function"*** (p. 113)

1. The resilience paradigm



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1. The resilience paradigm

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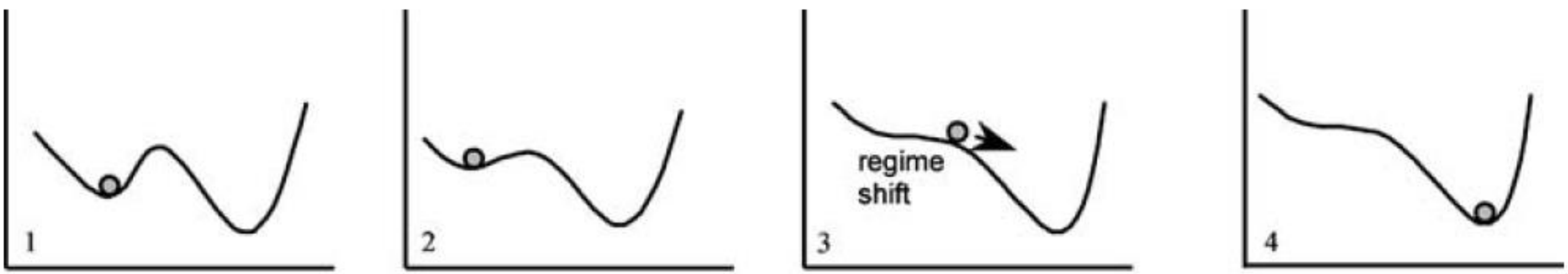
2. Defining resilience mathematically

resilience of **WHAT** to **WHAT** ?

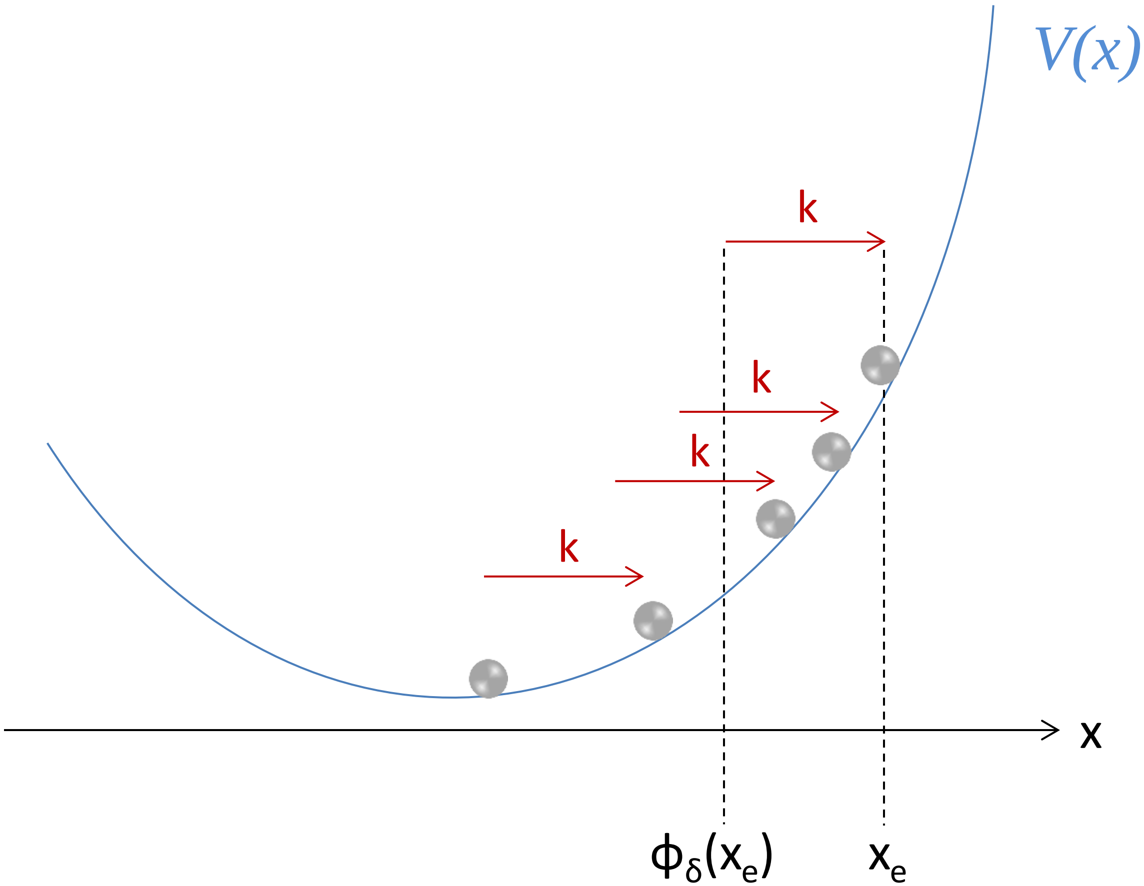
the basin of attraction

perturbation of…
- state variables
- parameters



Folke et al. (2004) *Annu. Rev. Ecol. Evol. Syst.*

3. A kick-flow system

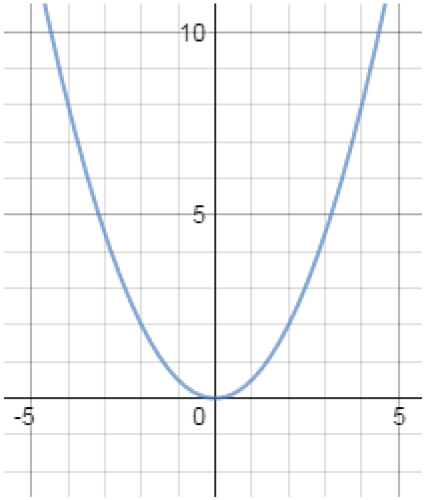


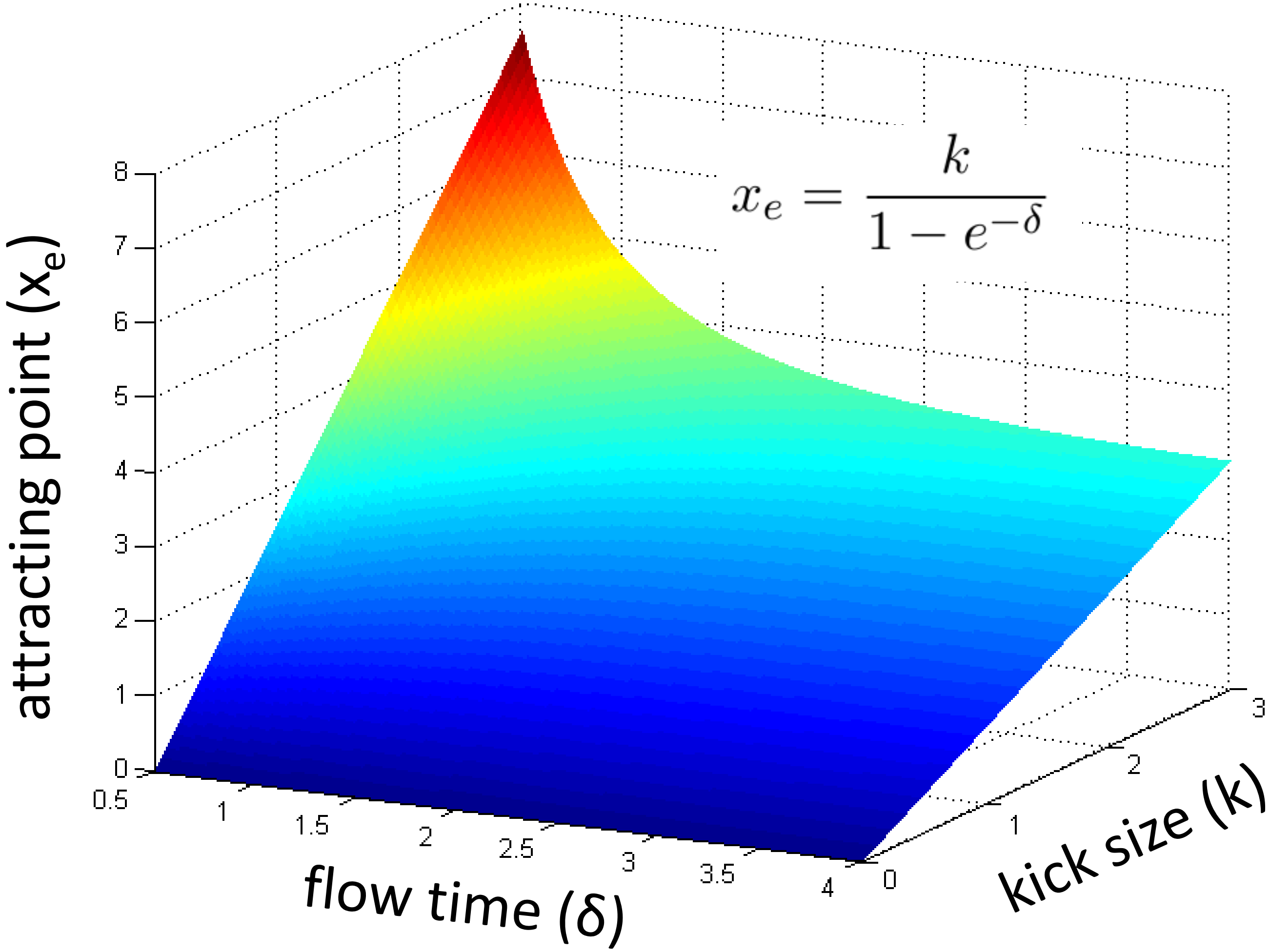
3. A kick-flow system

Linear 1D: Where is the attractor of this kick-flow system?

$$x'(t) = -x$$

$$V(x) = \frac{1}{2}x^2$$

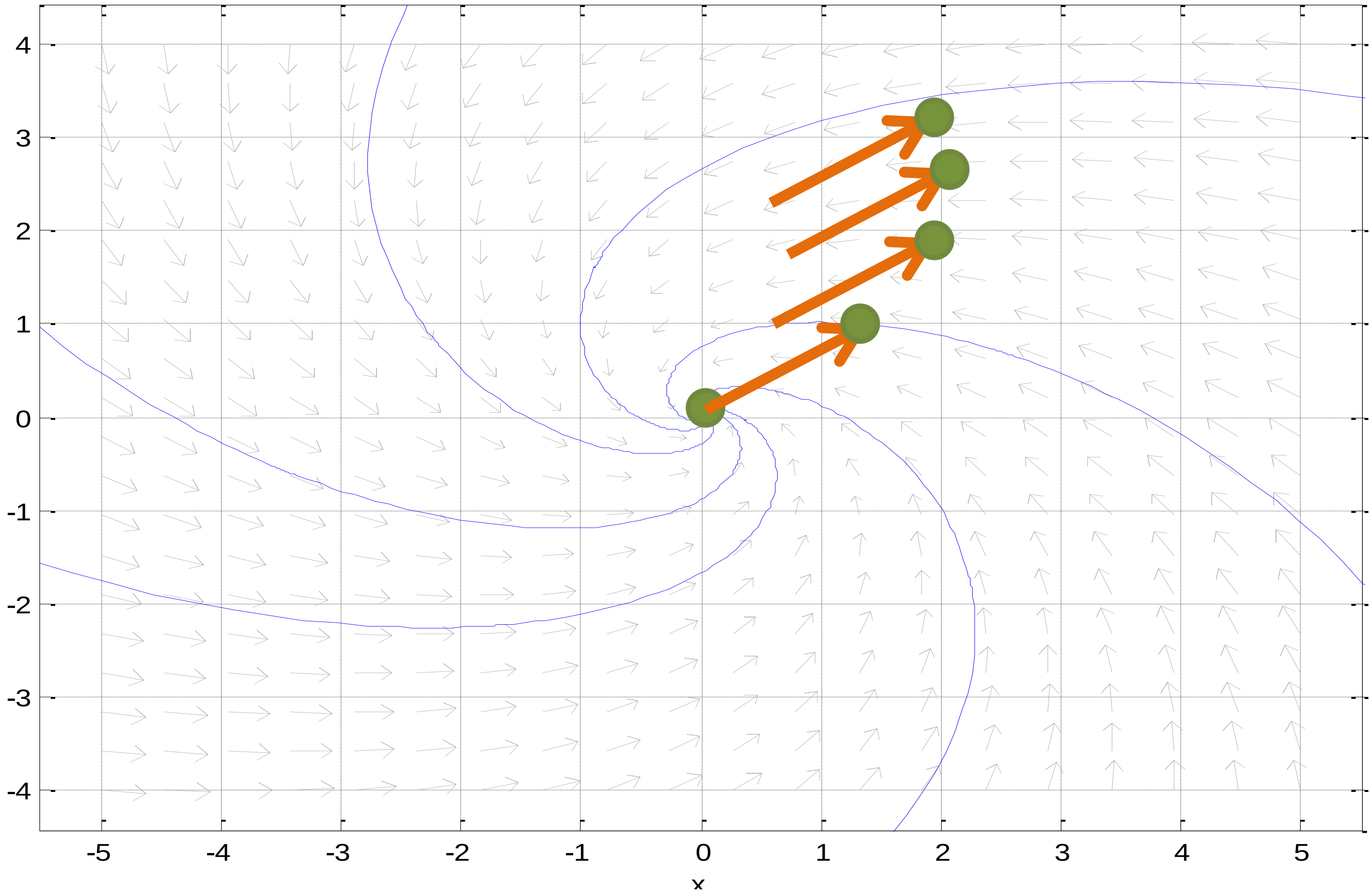




3. A kick-flow system

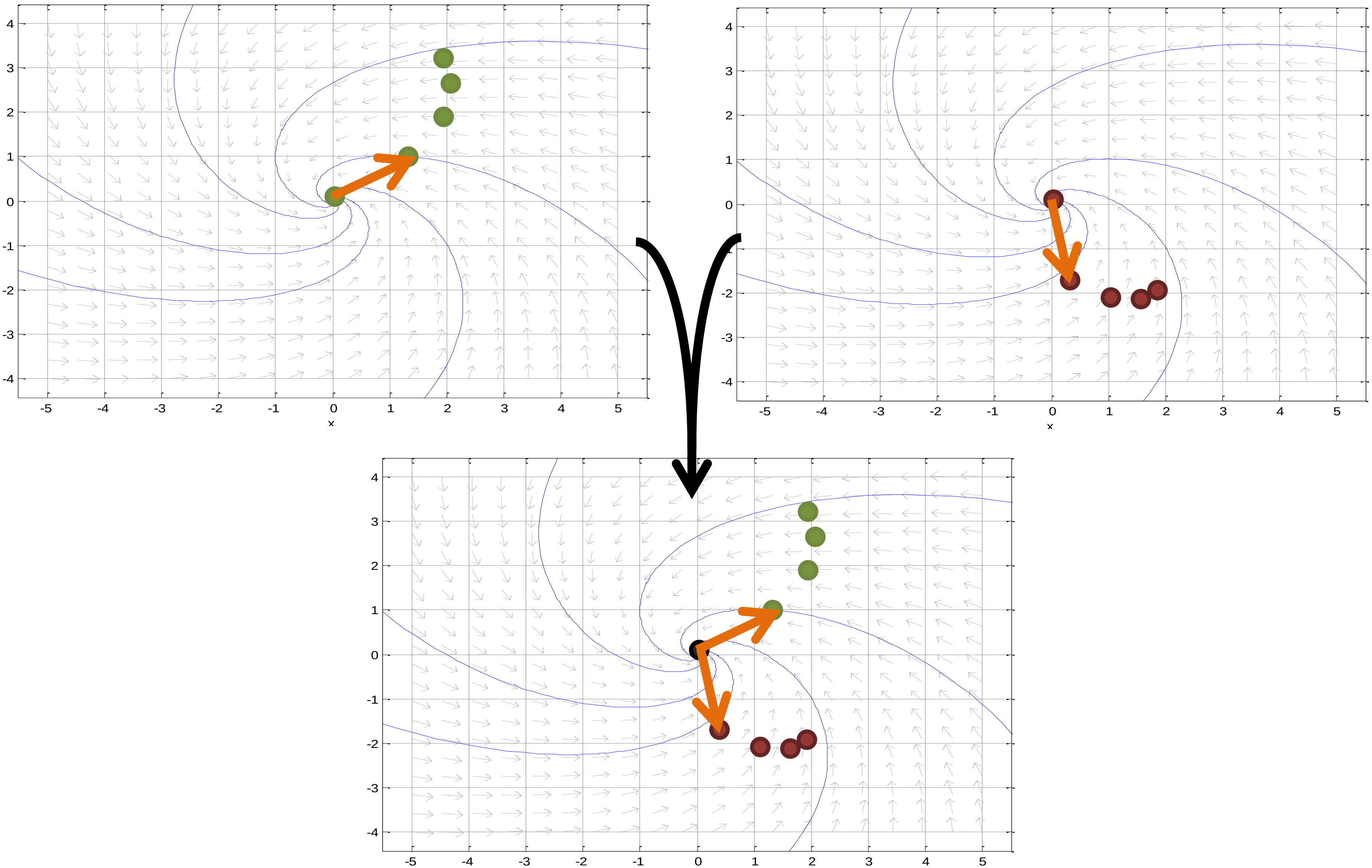
Linear 2D

$$\mathbf{X}' = \begin{bmatrix} -7/4 & -7/4 \\ 5/4 & -5/4 \end{bmatrix} \mathbf{X}$$



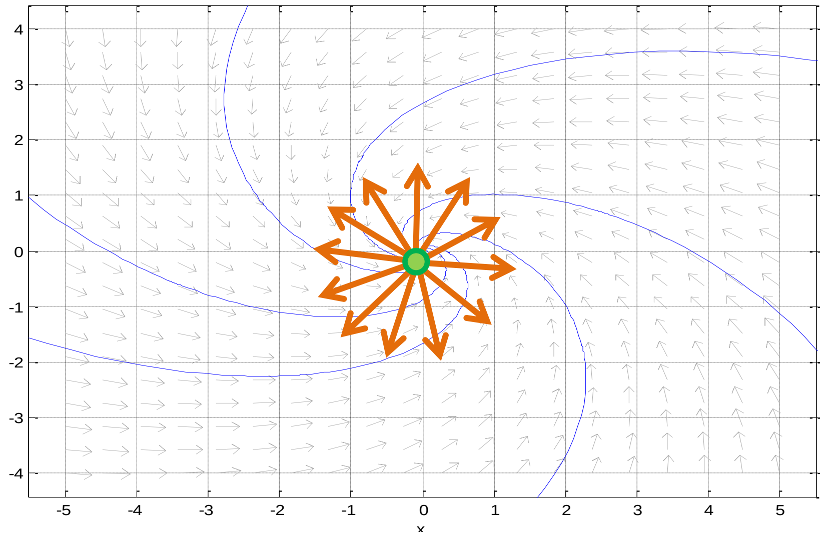
3. A kick-flow system

Linear 2D



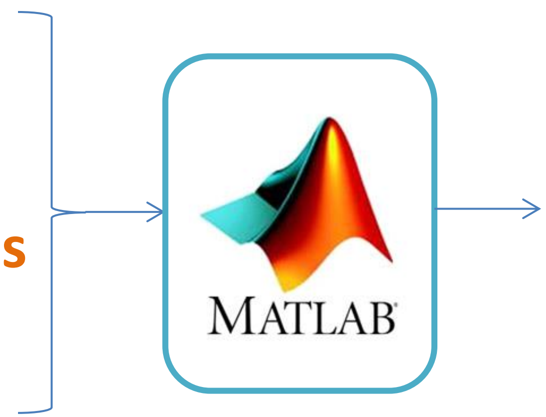
3. A kick-flow system

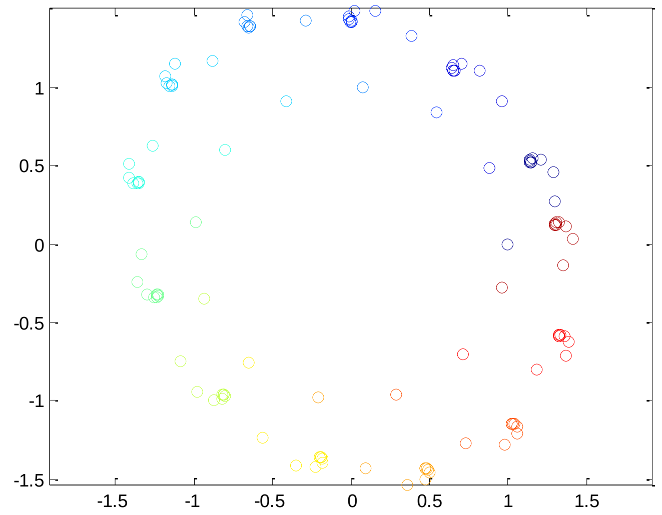
Linear 2D



Plot of discrete system

ODEs
initial position
kick size, directions
flow time



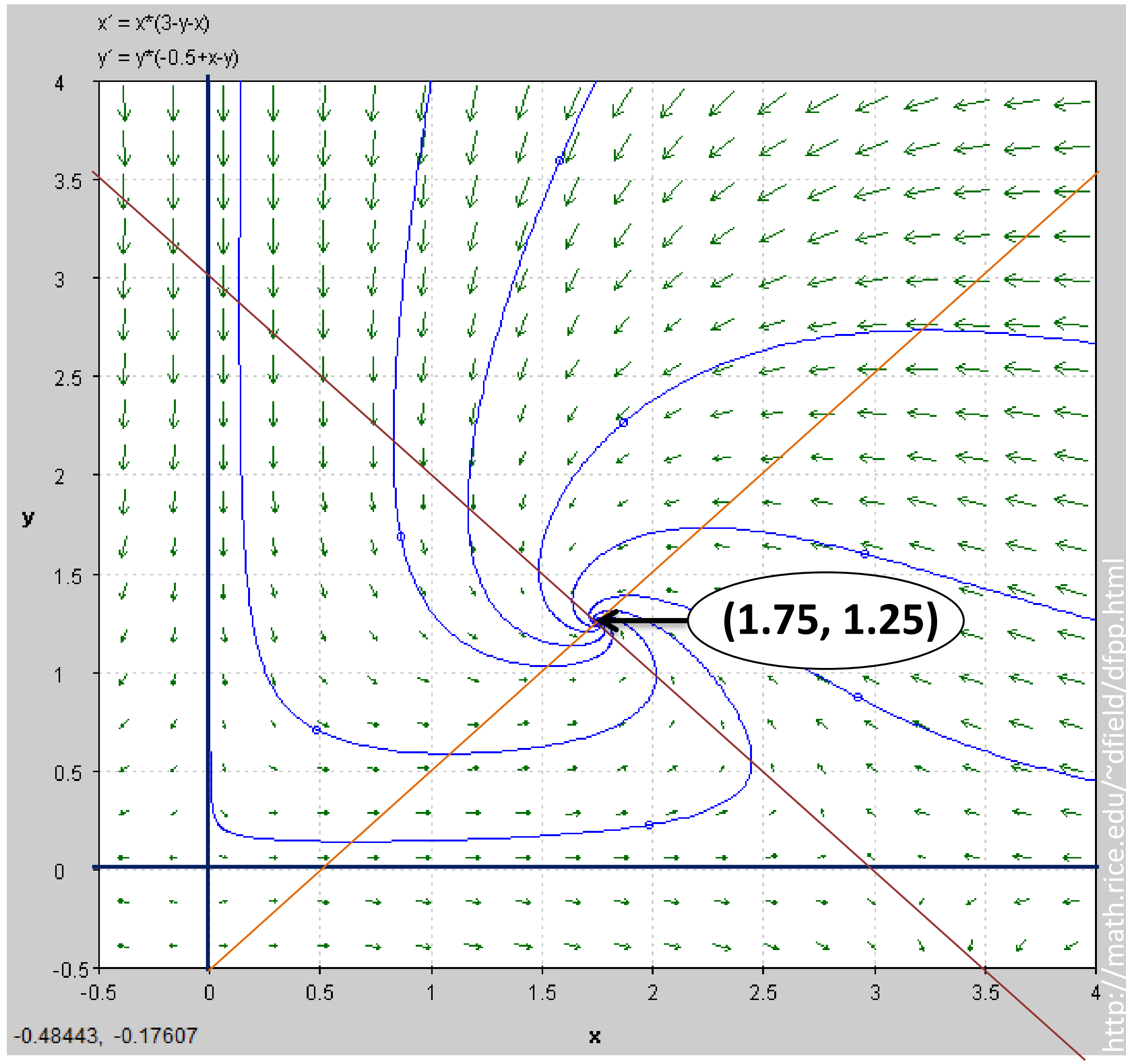


3. A kick-flow system

Nonlinear 2D

$$x'(t) = x(3 - y - x)$$
$$y'(t) = y(-0.5 + x - y)$$

Predator-Prey model for species with limited growth



3. A kick-flow system

Nonlinear 2D

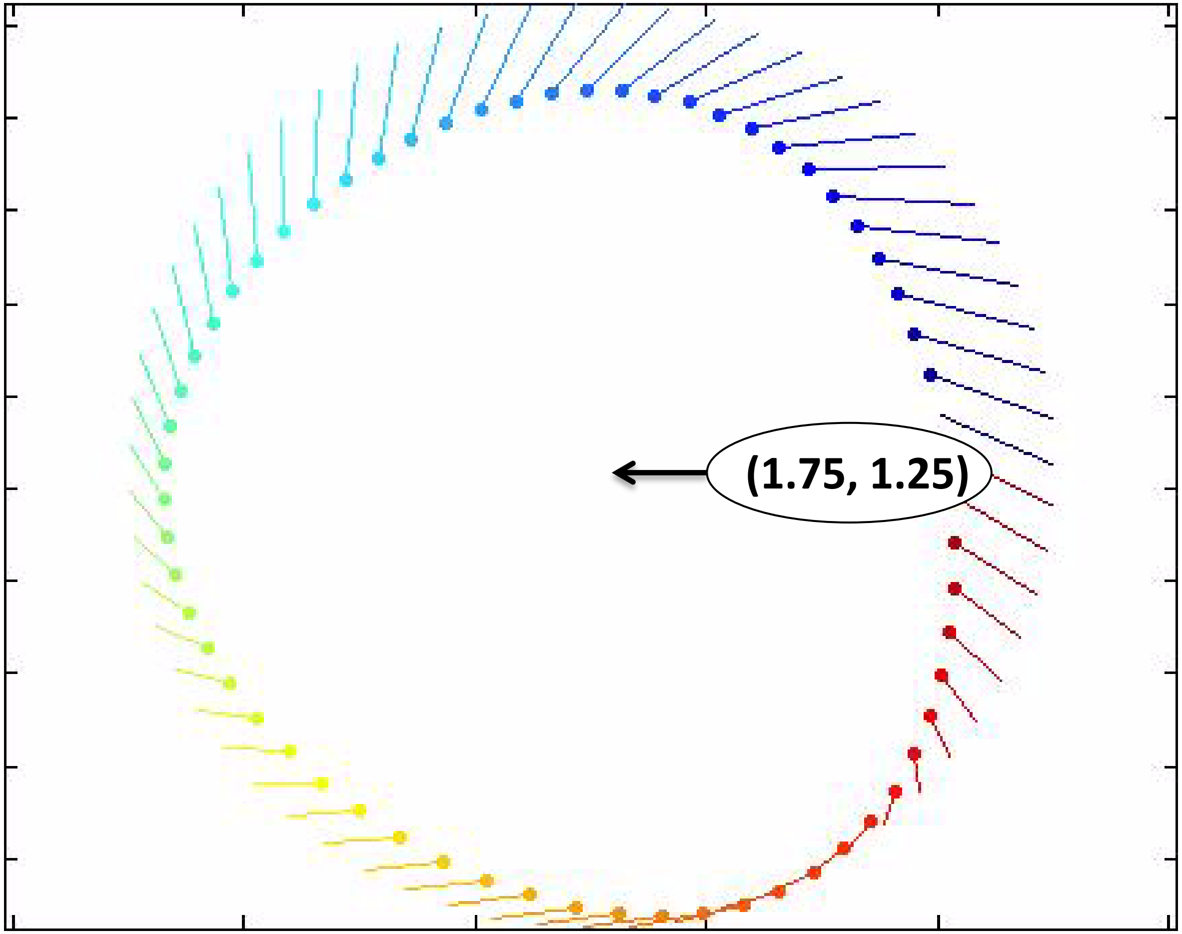
Predator-Prey model for species with limited growth

$$x'(t) = x(3 - y - x)$$
$$y'(t) = y(-0.5 + x - y)$$

kick size = 0

flow time = 0.1

iterations = 20



3. A kick-flow system

Nonlinear 2D

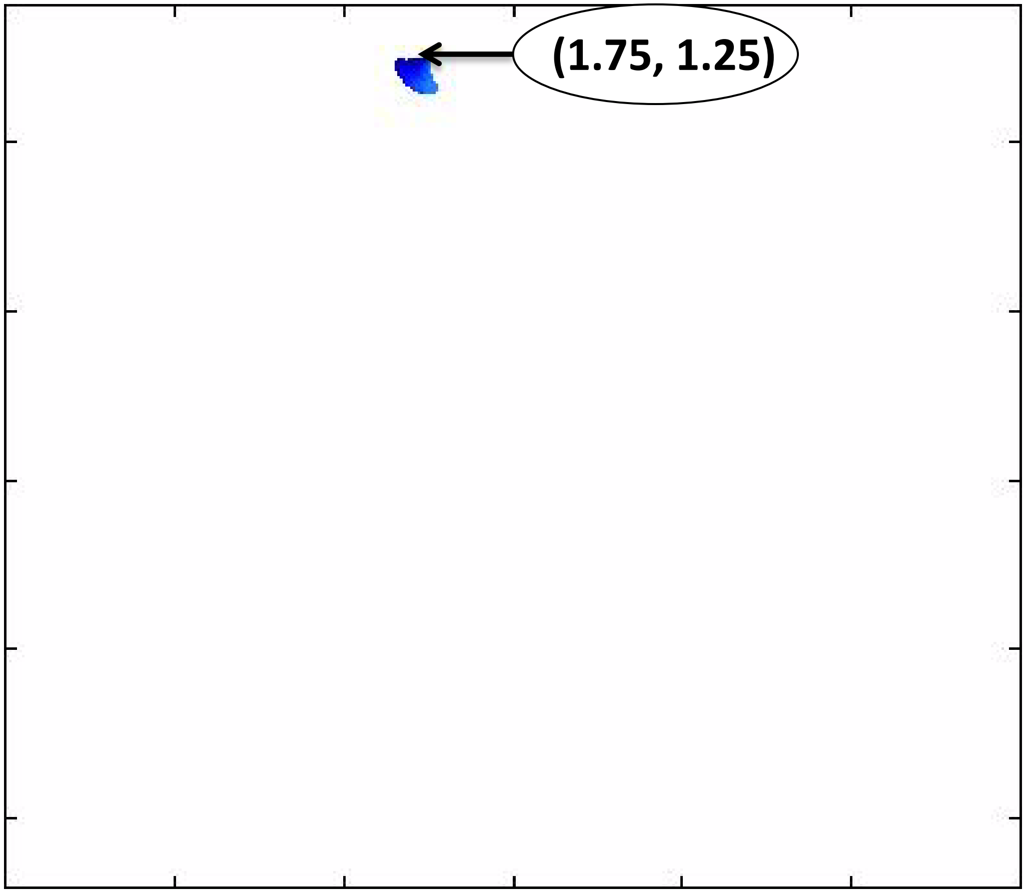
Predator-Prey model for species with limited growth

$$x'(t) = x(3 - y - x)$$
$$y'(t) = y(-0.5 + x - y)$$

kick size = 0.1
kick directions = π to $3\pi/2$

flow time = 0.1

iterations = 80



3. A kick-flow system

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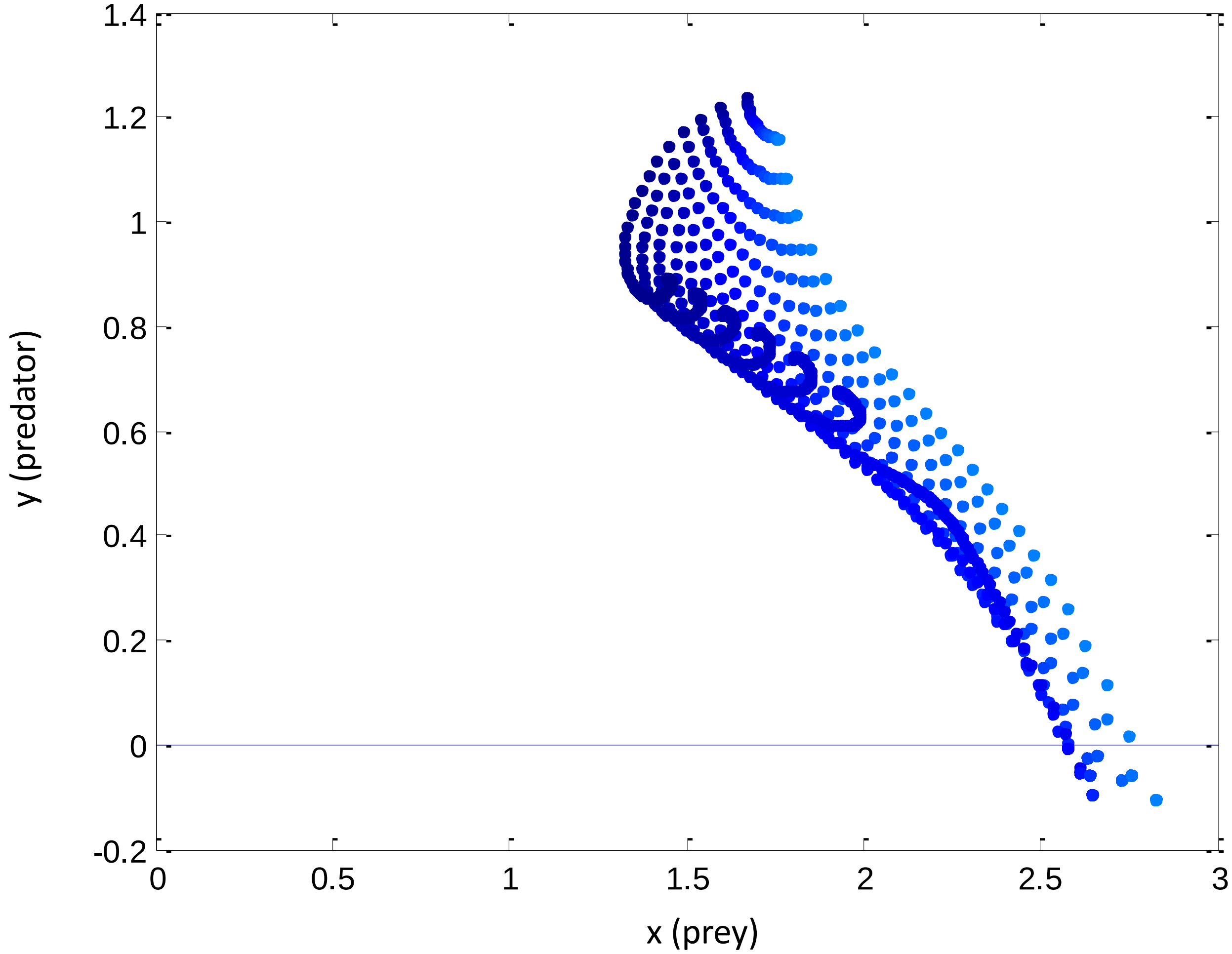


4. Future Directions

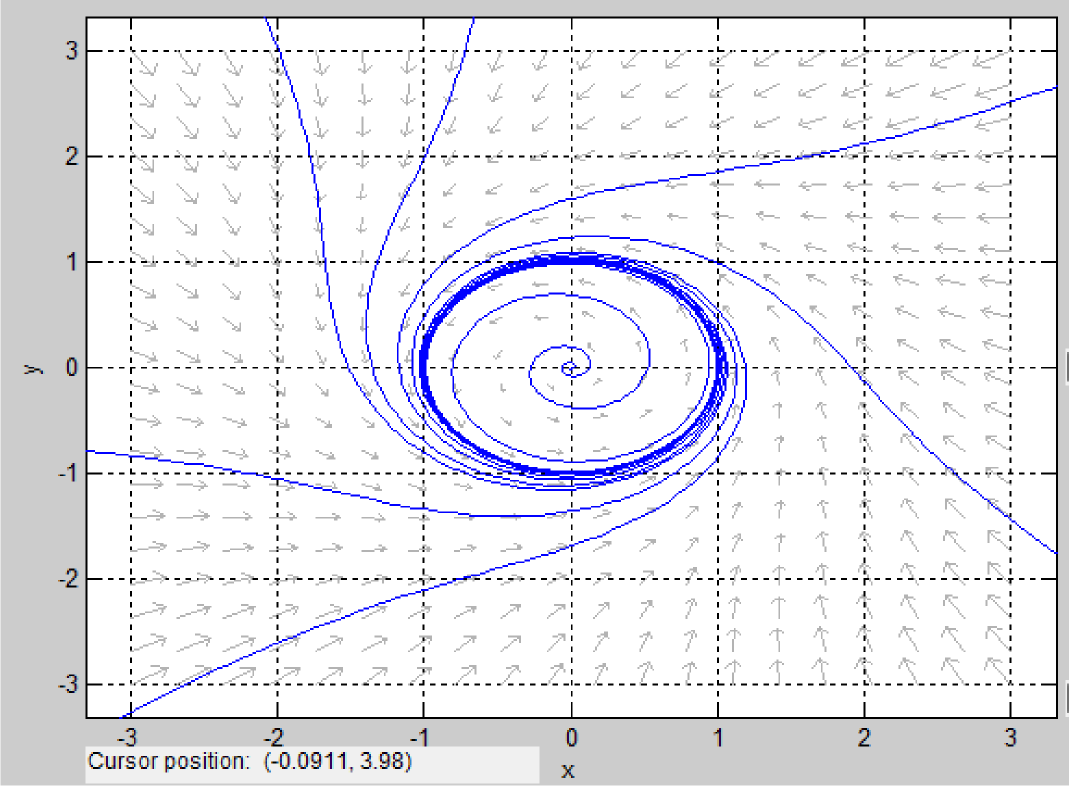
- Kick-flow in interesting 2D systems
  - attracting periodic orbit
  - excitable
  - unstable linear

- Modify kick-flow
  - change potential
  - multiple kick types
  - move towards stochastic kicks

- Your ideas?

4. Future Directions

Kick-flow in 2D system with attracting periodic orbit



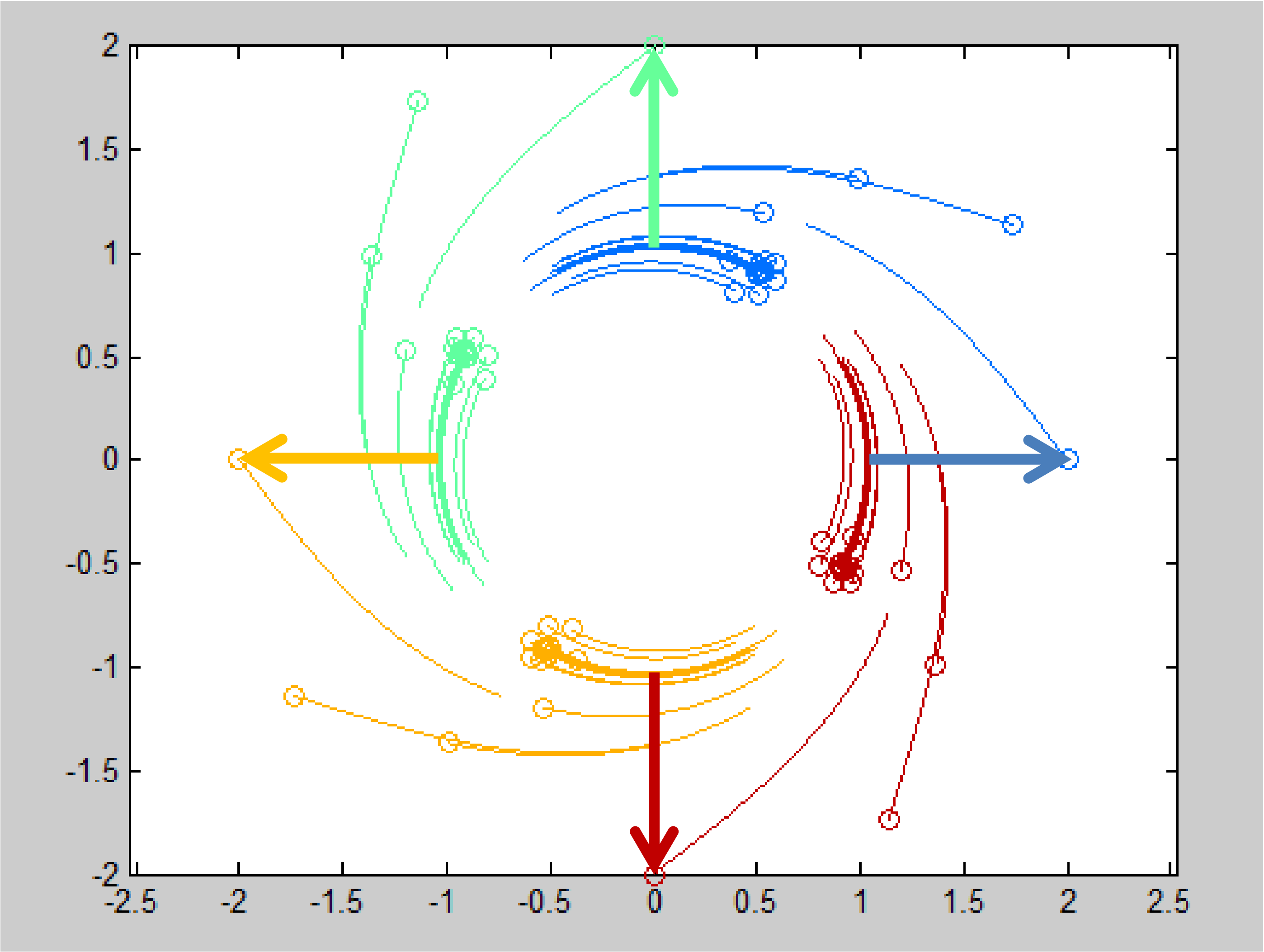
Hopf bifurcation normal form

$$r' = r(1 - r^2)$$
$$\theta' = 1$$

4. Future Directions

$$r' = r(1 - r^2)$$
$$\theta' = 1$$

kick size: 1
flow time: 1



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$$r' = r(1 - r^2)$$
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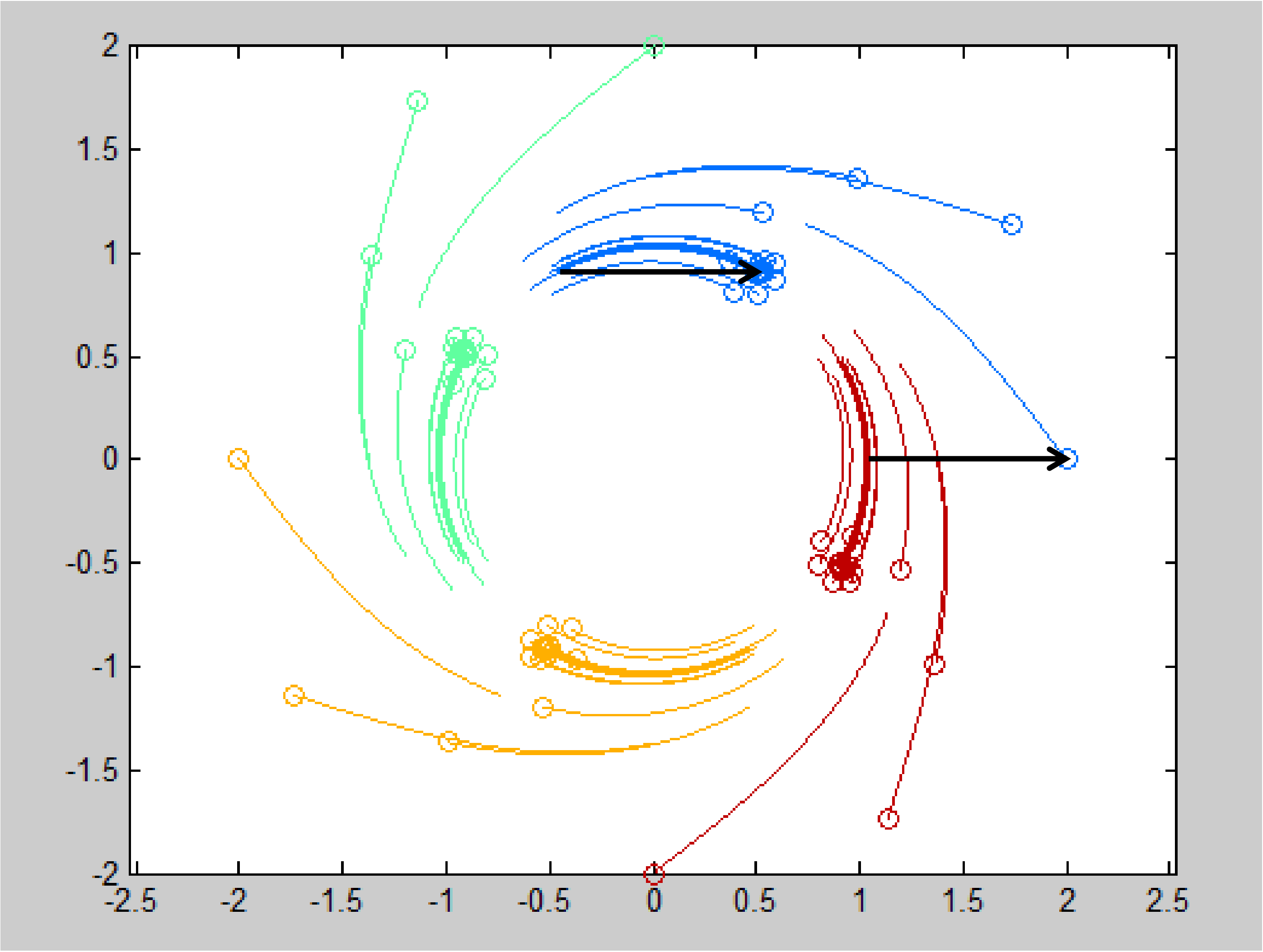
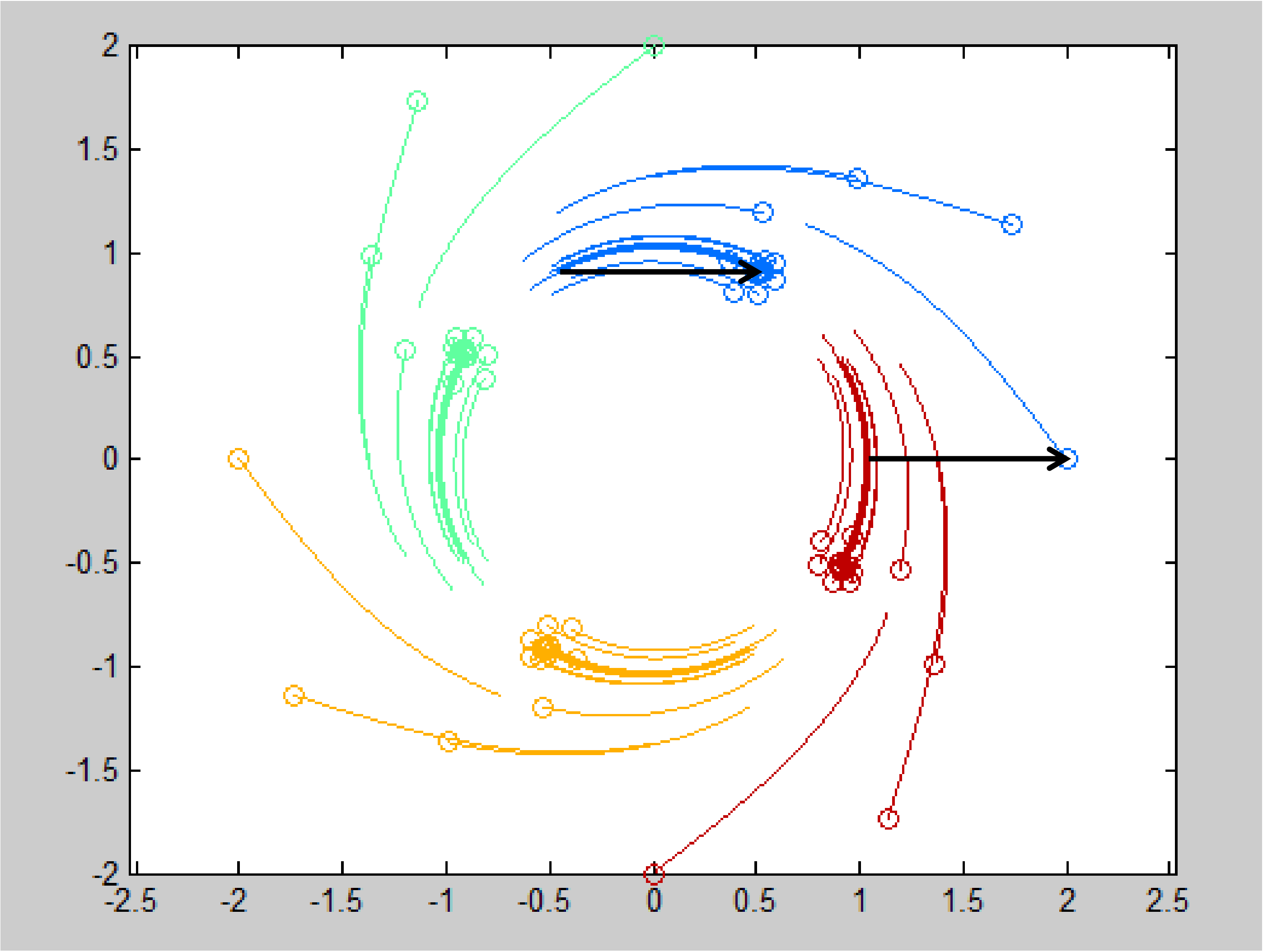


4. Future Directions

$$r' = r(1 - r^2)$$
$$\theta' = 1$$

kick size: 1
flow time: 1

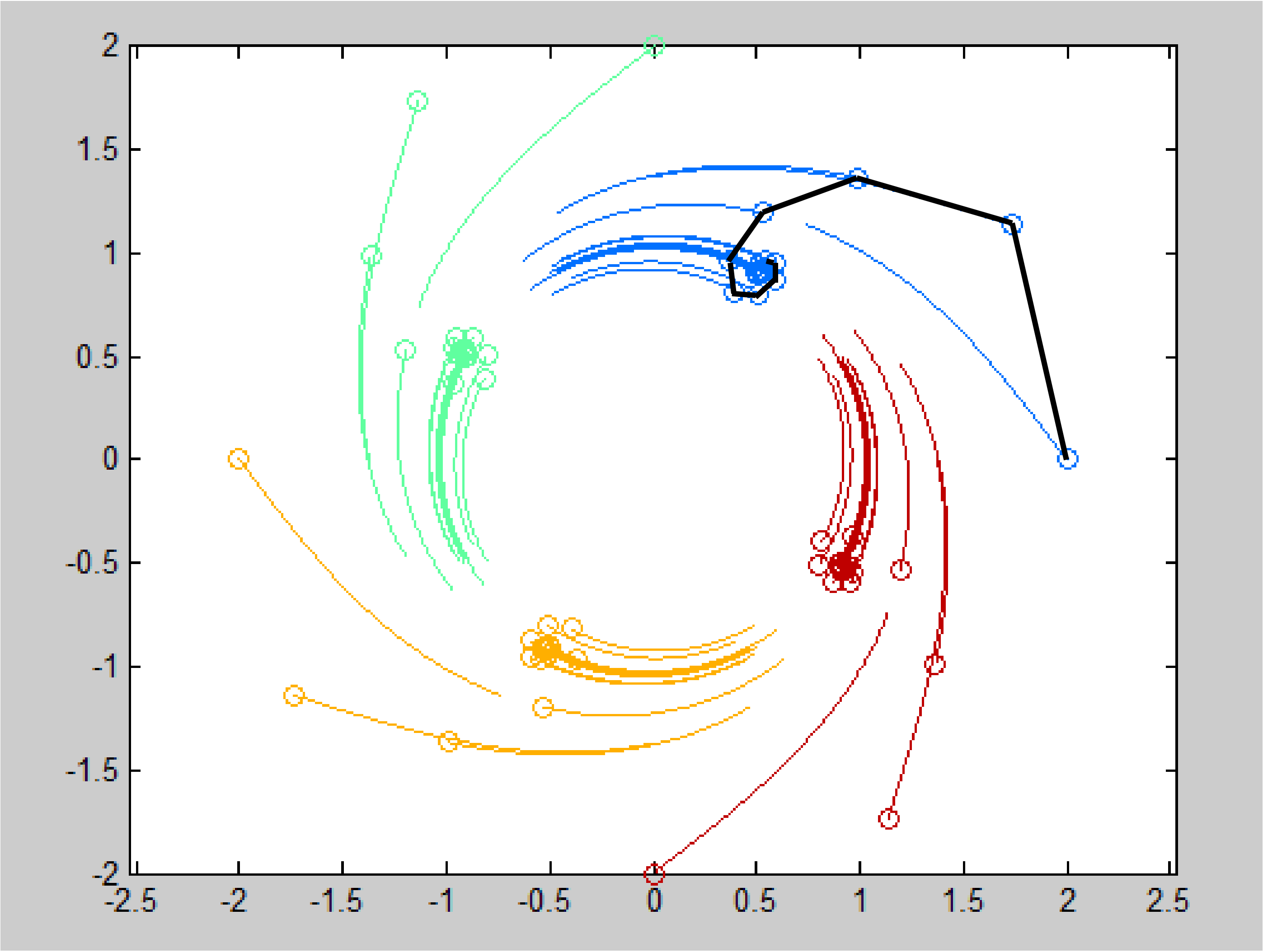


4. Future Directions

$$r' = r(1 - r^2)$$
$$\theta' = 1$$

kick size: 1

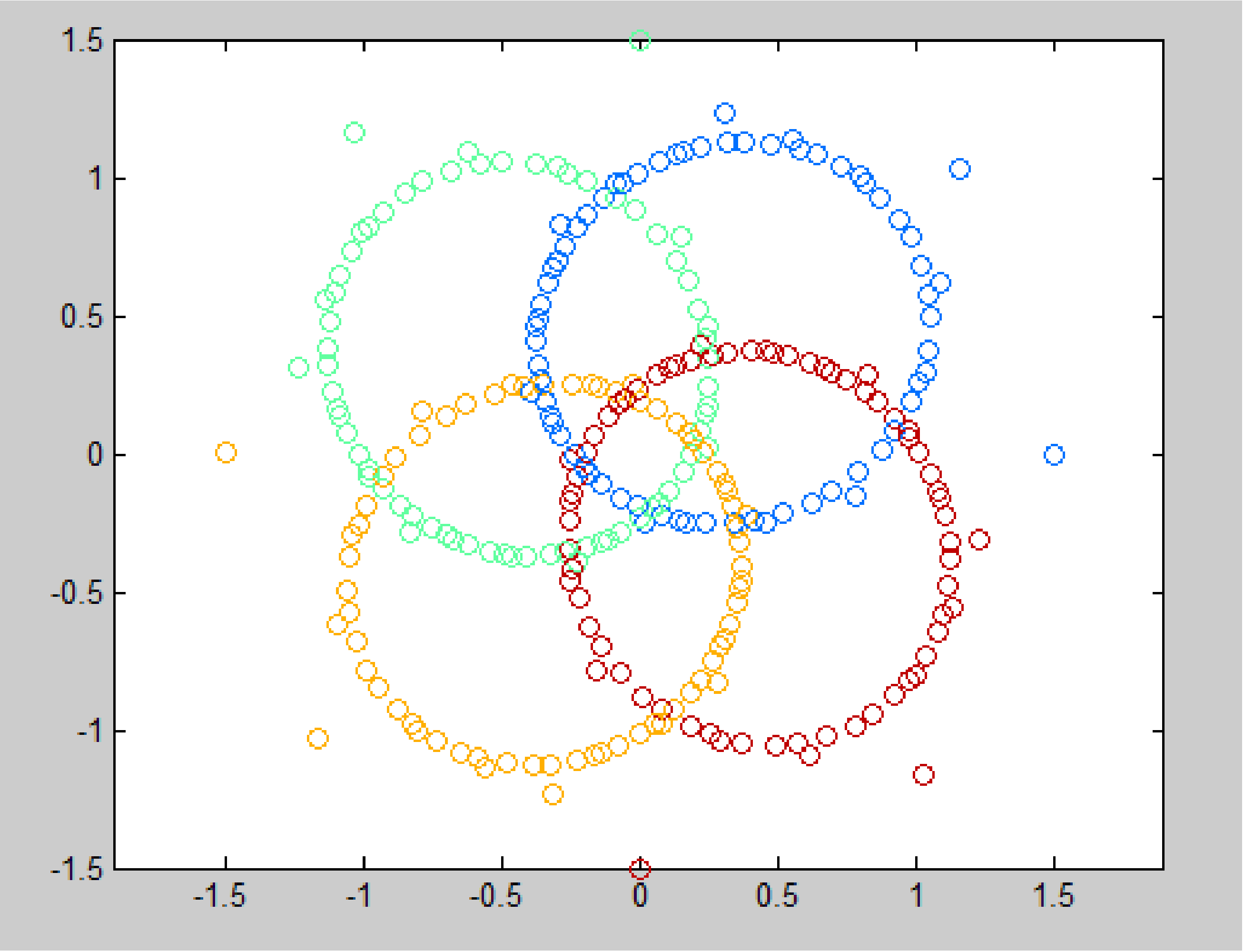
flow time: 1



4. Future Directions

$$r' = r(1 - r^2)$$
$$\theta' = 1$$

kick size: 0.5
flow time: 1



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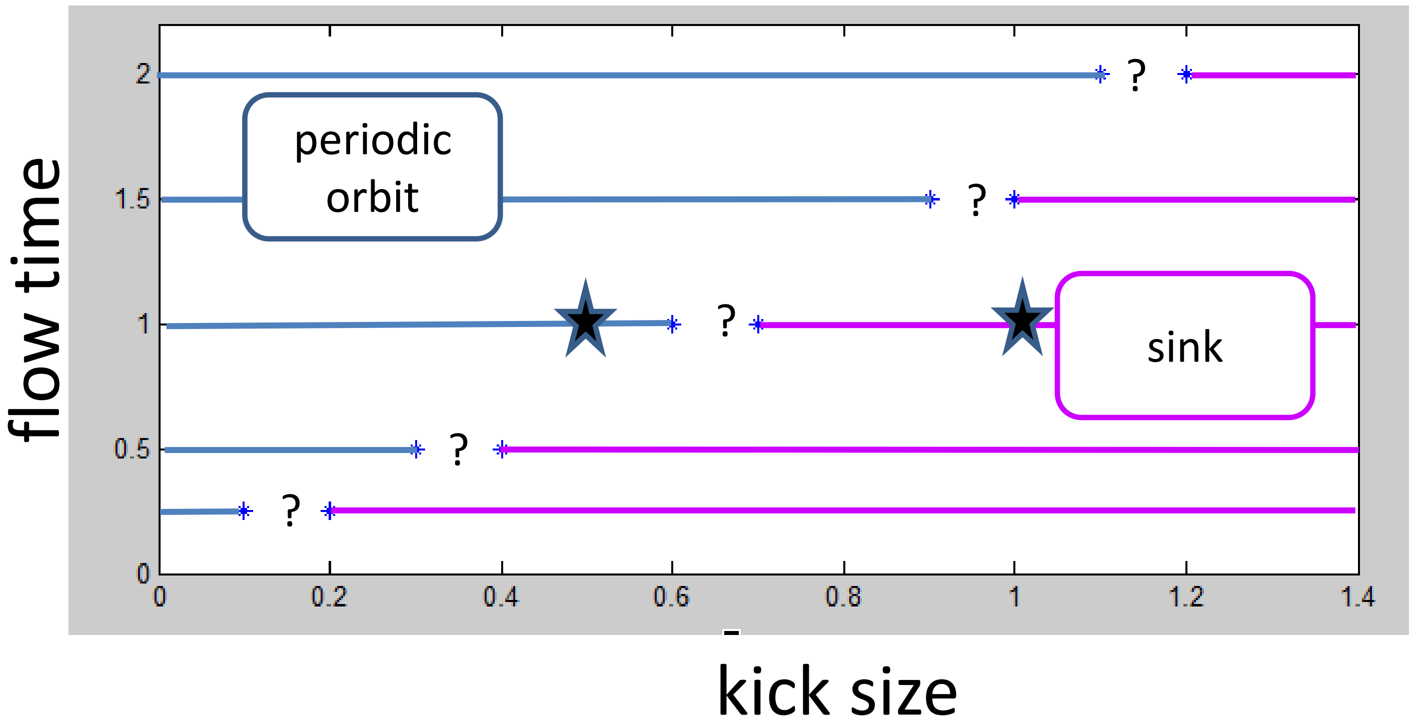

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Folke, Carl, et al. (2004) Regime shifts, resilience, and biodiversity in ecosystem management. Annual Review of Ecology, Evolution, and Systematics 35: 557-581.

Frelich, Lee, and Peter Reich. (2009) Will environmental changes reinforce the impact of global warming on the prairie-forest border of central North America? Frontiers in Ecology and the Environment 8: 371–378.

Mumby, Peter, et al. (2007) Thresholds and the resilience of Caribbean coral reefs. Nature 450: 98-101.

Walker, Brain and David Salt. (2006) *Resilience thinking: sustaining ecosystems and people in a changing world*. Washington DC: Island Press. Print.

Walker, Brian, et al. (2004) Resilience, adaptability and transformability in social-ecological systems. Ecology and Society 9:5.

Details on kick-flow-equilibrium plot for 1d linear system

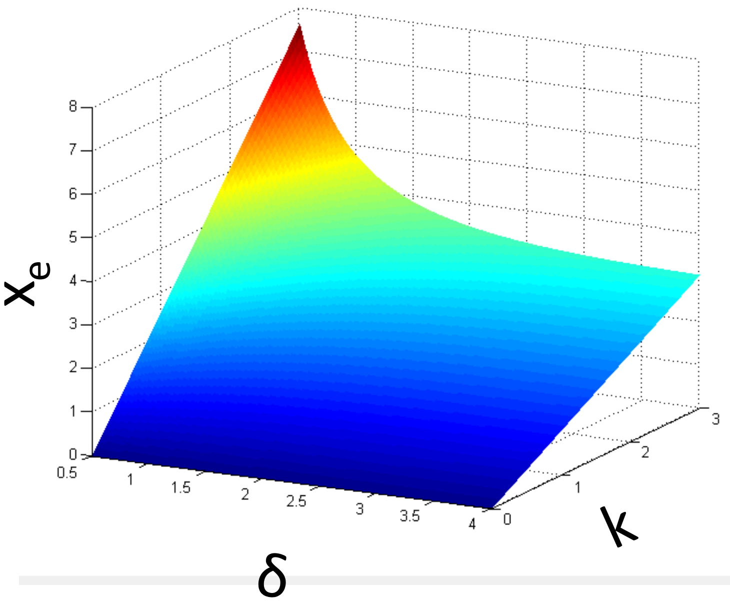
Calculation of $x_e(\delta, k)$

$$x' = -x$$
$$\phi_t(x) = xe^{-t}$$

want $\quad \phi_\delta(x_e) = x_e - k,$

so $\quad x_e e^{-\delta} = x_e - k$

$$\boxed{\implies x_e = \frac{k}{1 - e^{-\delta}}}$$



MATLAB commands (a=1):

```
>> [D,K]=meshgrid(0.5 : 0.02 : 4, 0 : 0.02 : 3);
>> C=K./(1-exp(-D));
>> surf(D,K,C,'EdgeColor','none')
```